AD-A278 597

MENTATION PAGE

Form Approved
OMB No. 0704-0188

2

REPORT DATE

3. REPORT TYPE AND DATES COVERED
FINAL/01 AUG 90 TO 30 SEP 93

4. TITLE AND SUBTITLE
COMPUTATIONAL MATHEMATICS LABORATORY
FOR MULTISCALE ANALYSIS (U)

5. FUNDING NUMBERS
9806/DARPA
AFOSR-90-0334

6. AUTHOR(S)
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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
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8. PERFORMING ORGANIZATION REPORT NUMBER
AFOSR-TR- 94 0249

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AFOSR/NM
110 DUNCAN AVE, SUITE B115
BOLLING AFB DC 20332-0001

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
AFOSR-90-0334

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
APPROVED FOR PUBLIC RELEASE: DISTRIBUTION IS UNLIMITED

12b. DISTRIBUTION CODE
UL

13. ABSTRACT *(Maximum 200 words)*

The research done by the Computational Mathematics Laboratory (CML) at Rice University with the support of ARPA and AFOSR Grant. The principal research activity was; (1) Fundamental Wavelet Research, (2) Applications of Wavelets to Partial Differential Equations, (3) Applications of Wavelets to Digital Signal Processing.

14. SUBJECT TERMS

15. NUMBER OF PAGES
19

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAR(SAME AS REPORT)

AEOSR-TR- 94 0249

Research Activities of the Computational Mathematics Laboratory, Rice University, 1990-1993

Final Technical Report

February 11, 1994

1 Introduction

This report represents the final report to ARPA and AFOSR concerning the research done by the Computational Mathematics Laboratory (CML) at Rice University with the support of ARPA by means of a grant administered by AFOSR (Grant No. 90-0334).

The principal research activity concerned the following three areas:

- Fundamental wavelet research
- Applications of wavelets to partial differential equations
- Applications of wavelets to digital signal processing

The details of the research are contained in 69 technical reports which are listed at the end of this report and which are available from the laboratory. The vast majority of these reports have been or are in the process of being published as indicated in the listing.

This report will summarize the main results from the point of view of the differential equations group and the digital signal processing group, both of which have made significant contributions to the area of fundamental wavelet research, and specifically to these application areas, respectively.

The principal results in basic wavelet analysis concern the extension of wavelets from multiplier 2 to higher rank (or larger M for M-band wavelets to use the signal processing language). This includes primarilty the parametrization of such wavelet systems, and the special theory of cosine-modulated wavelets, as well as many other special families. Basic phenomena of sampling, interpolation, optimization, and modeling have all been considered in the wavelet context.

In the context of the study of differential and integral equations a number of results have been established concerned with the representation of differential and pseudodifferential operators

and with the solutions of boundary value problems. The basic principle of fictitious domains has been used as a tool to translate a boundary value problem to an integral equation on a larger rectangular region in which the boundary data and the geometry of the boundary become encoded in the inhomogeneous term of the resulting discretized linear and nonlinear algebraic equations (using the wavelet-Galerkin discretization principle).

The basic problems which were solved by this methodology include:

- Solving Dirichlet and Neumann problems at a single scale for linear and nonlinear elliptic boundary value problems with very general boundary where the basic coding is independent of the geometry of the boundary
- Solving Dirichlet problems using a wavelet-based multilevel preconditioner for a conjugate-gradient iteration method which is far superior to a normal conjugate-gradient method
- Formulating and solving a highly singular anisotropic model differential equation with periodic boundary conditions with a wavelet multigrid algorithm which yields an iteration matrix with a spectral radius smaller than 1 and which is independent of the mesh size and the anisotropy parameter

The main results of the research on signal processing using wavelets include:

- Methods for optimizing the family of wavelet basis functions were developed to allow tailoring the wavelets to the particular signals being analyzed.
- A theory and set of tools for using time-varying wavelets has been developed. This allows changing the nature of the basis functions during an analysis while maintaining all of the properties of the wavelets.

2 Summary of Partial Differential Equations Research: R. O. Wells, Jr., Andreas Rieder and Xiaodong Zhou

During the year 1993 the CML investigated the fast and efficient resolution of a class of linear systems arising by a wavelet-Galerkin discretization of the following elliptic model problem

$$-\alpha\, u_{xx} - \beta\, u_{yy} + u = f \quad \text{in} \quad \omega \in \mathbf{R}, \tag{1}$$

$$u = g \quad \text{on} \quad \partial\omega, \tag{2}$$

via a penalty/fictitious domain formulation introduced e.g. in [8], [38] and [39]. The constants α and β in (1) are positive and ω is an open domain in $\mathbf{R}^2$ (however, there is no principal restriction to two dimensions).

The penalty/fictitious domain formulation of the boundary-value problem (1), (2) becomes: let $\Omega = [0, R]^2$, $\omega \subset \Omega$, be the fictitious domain and $\epsilon > 0$ the penalty parameter. Then, we seek a $u^\epsilon \in H_p^1(\Omega)$, the Sobolev space of order 1 with periodic boundary conditions on Ω, such that

$$\int_\Omega (\alpha u_x^\epsilon v_x + \beta u_y^\epsilon v_y + u^\epsilon v) dx dy + \frac{1}{\epsilon}\int_{\partial\omega} u^\epsilon v \, ds = \int_\Omega \tilde{f} v \, dx dy + \frac{1}{\epsilon}\int_{\partial\omega} g v \, ds, \tag{3}$$

for all $v \in H_p^1(\Omega)$, where, in (3), $\tilde{f}$ is an arbitrary L^2-extension of f in Ω. The solution u^ϵ converges to $\tilde{u}$ in $H^1(\Omega)$ for $\epsilon \to 0$, where $\tilde{u}$ is the $H^1(\Omega)$-extension of the solution of the following variational problem: $\tilde{u} \in H_p^1$, $\tilde{u} = g$ on $\partial\omega$,

$$\int_\Omega (\alpha \tilde{u}_x v_x + \beta \tilde{u}_y v_y + \tilde{u}\, v) dx dy = \int_\Omega \tilde{f}\, v \, dx dy$$

for all $v \in H_p^1$, such that $v = 0$ on $\partial\omega$.

Using the Daubechies scaling functions φ of order $N \geq 3$, [2], we define the periodic wavelet-Galerkin space at level L by ($R \in \mathbf{N}$)

$$V_L^p = V_L^p(0, R) := \{v \in L^2(0, R) \;\; : \;\; v(x) = \sum_{k \in \mathbf{Z}} c_k \varphi_k^L(x),\ x \in [0, R], \\ \text{with } c_k = c_{k+2^L R}\}$$

with $\varphi_k^L(x) = 2^{L/2}\varphi(2^L x - k)$, and we approximate H_p^1 by the tensor product $X_L = V_L^p \otimes V_L^p$. The penalty problem (3) restricted to X_L is equivalent to the linear system,

$$A_{L,\epsilon} U_L^\epsilon = f_L + \epsilon^{-1} M_L g_L, \tag{4}$$

see [9], [39], with appropriate right hand sides $f_L, g_L \in \mathbf{R}^n$, $n = n_L = \dim V_L^p = 2^L R$. The stiffness matrix $A_{L,\epsilon}$ is of the form $A_{L,\epsilon} = A_L + \epsilon^{-1} M_L \in \mathbf{R}^{n\times n}$, where A_L is symmetric and positive-definite. The diagonal matrix M_L corresponding to the boundary integrals in (3) has the diagonal entries either 1 or 0.

The presence of the penalty term $\epsilon^{-1} M_L$ in (4) requires special attention in order to achieve an efficient solver. Therefore, we first reduce the influence of the penalty parameter ϵ. The family of solutions $\{U_L^\epsilon\}_{\epsilon>0}$ of (4) has the limit U_L which is given by

$$U_L = \xi^* + M_L g_L,$$

where ξ^* is the unique solution of

$$(I - M_L)A_L(I - M_L)\xi = (I - M_L)(f_L - A_L M_L g_L) \tag{5}$$

in the range $R(I - M_L)$ of $(I - M_L)$, see [9]. Instead of the system (4) we choose to solve (5). Since $(I - M_L)A_L(I - M_L)$ is symmetric and positive-definite on $R(I - M_L)$ we can use the conjugate gradient method (*cg*-method) for the iterative solution of (5).

A natural choice for a preconditioner of the *cg*-method acting on (5) is any symmetric iteration for the fast solution of linear systems with matrix A_L. The latter matrix can be interpreted as the stiffness matrix of the following variational problem: find $u \in X_L$, such that

$$\int_\Omega (\alpha u_x v_x + \beta u_y v_y + u\,v)dxdy = \int_\Omega w\,v\,dxdy \tag{6}$$

for all $v \in X_L$.

For the variational problem (6) we developed multilevel methods. In the isotropic case ($\alpha \approx \beta$) the step-size independent convergence rate of these methods can be proved by techniques closely related to those used for finite difference and finite elemente discretizations, see [9].

Things become more complicated in the anisotropic case ($\alpha \ll \beta$ or $\alpha \gg \beta$). Here, we developed a wavelet variation of the frequency decomposition multigrid method of Hackbusch [26], [27], see [35]. This iteration is robust, that is, the convergence speed is not only independent of the discretization step-size but also of the parameters α and β, see [36].

Various numerical experiments presented in [9] show the efficiency of the multilevel methods for (6) used as preconditioners for the *cg*-method applied to (5). However, an analytic statement remains to be established and shall be considered in future research projects.

3 Summary of Research at the University of Houston: R. Glowinski and T. W. Pan

Our work supported by ARPA during these last three years has been oriented to the following two major directions:

(i) Wavelet Approximation of Incompressible Viscous Flow

(ii) Fictitious Domain Methods for Partial Differential Equations.

Concerning **(i)**, we think that we have fully elucidated the always delicate issues about the compatibility conditions between the pressure and velocity approximations. From that point of

view, wavelets are ideally suited to address these issues since if δ is the smallest length scale used for the velocity approximation, we should use similar scaling and wavelet functions to approximate the pressure space, but with 2δ as smallest scale. Indeed this wavelet motivated analysis has been also useful to better understand compatibility conditions between finite element spaces used to approximate viscous flow problems and also problems from Control Theory. Indeed, in [4] we have explored the analogy between these various problems and shown how these considerations apply to wavelet approximations.

Concerning **(ii)**, we have combined fictitious domain methods to wavelet approximations to obtain robust wavelet solution methods for various classes of elliptic problems. Also, these fictitious domain methods in which one tries as much as possible to decouple the approximation of the actual geometry from the approximation of the imbedding space have proved very useful for finite element methods and have provided tools allowing the solution of problems in nonregular geometries via the use of regular means. Our first investigations and numerical tasks were initially concerned with simple elliptic problems such as Neumann ([5]) and Dirichlet ([6]). Since then, these methods have been applied to the solution of incompressible viscous flow problems ([7]) and very recently to the solution of scattering problems by obstacles of nonregular shape. ([10]). These methods initially developed and tested with finite element approximations are presently investigated in order to develop wavelet implementations, including implementations on parallel machines.

4 Summary of Research: W. W. Symes and G. Bao

Prof. Symes and postdoctoral associate Dr. Gang Bao worked on the representation of pseudodifferential operators. Various classes of these operators have been discussed in the wavelet literature, and indeed singular integrals on the line have been shown to be well-approximated with sparse matrices in wavelet bases, in work of Meyer, Beylkin, Coifman, Rohklin, Jaffard, and others. However multidimensional nonseparable pseudodifferential operators did not seem to have been investigated in this regard, but form the class most useful in wave propagation theory, Prof. Symes' long-term interest.

Our work reached two basic conclusions. First, at least tensor product wavelet bases do not produce particularly sparse representations of these operators. Second, we used the representation of the pseudodifferential operators as the algebra generated by differential operators and all powers of the Laplacian to derive an efficient Fourier transform based algorithm for evaluation of the operator action. We implemented this algorithm in MATLAB and in CM Fortran, and tested it

successfully. It evaluates the action of an operator in d diminsions on a function represented on a grid with N^d gridpoints in $O(N^d \log N)$ operations. It is hard to see how another algorithm, whether based on wavelet or some other technology, could have a more favorable asymptotic complexity.

5 Summary of Signal Processing Research: R. A. Gopinath, J. E. Odegard, C. S. Burrus and H. Guo

5.1 Introduction

This section contains a summary of the work done at the Computational Mathematics Laboratory (CML) at Rice University by the Digital Signal Processing (DSP) group during the period 1990-1993. The main thrust of the research has been toward developing a theory for wavelet analysis in signal processing and in particular the development of relations between wavelet theory and filter bank theory. A detailed presentation of the work can be found in the numerous papers and technical reports written by members of the DSP group (see the references to papers and technical reports by Burrus, Gopinath, and Odegard appended). Concurrent with the theoretical development we have also worked on developing a Matlab toolbox for wavelet design and analysis "`rice-wlet-tools`" which is available via anonymous ftp from "`cml.rice.edu`" in the directory "`pub/dsp/software`". In the following paragraphs we will give a brief outline of various projects which have contributed to the main thrust of our research.

5.1.1 Time-Varying M band Multiscale Analysis

Recently it was discovered that time-varying design/analysis could be associated with the multiresolution concept. The time-varying concept generates a framework for performing "adaptive" signal dependent wavelet analysis and subband analysis. Research on time-varying filter banks and wavelet multiresolution in the DSP group [22,12,11] has been on the development of a complete factorization of all optimal (in terms of quick transition) time-varying FIR unitary filter bank tree topologies. This has potential applications in areas such as adaptive subband coding, adaptive tiling of the time-frequency plane, and the construction of orthonormal wavelet bases for the half-line and interval [30,29,33,3]. A simple efficient implementation algorithm also comes with the factorization ensuring that even the most complex tree topology can be adapted with minimal overhead. Explicit formulas for transition filters/functions are derived for arbitrary tree transitions. The results are independent of the number of channels and the length of the filters (as long as they are FIR), implying that some of the efficiency reasons for considering only binary time-varying trees is not

valid any more. Time-varying wavelet bases (different bases for different segments of the real line) are also constructed.

5.1.2 Flexible M-band Multiscale Analysis

Wavelet analysis gives a flexible method for the analysis of non-stationary signals. One can simultaneously analyze short-duration wide-band signals and long-duration narrow-band signals. However, the traditional 2-band wavelets cannot be used to analyze signals like a long-duration RF pulse. To overcome this problem we have introduced M-band wavelet frames and wavelet tight frames. By specializing on a well-known parameterization of unitary filter banks we have obtained a complete parameterization of compactly supported M-band wavelet bases (see [18,37] and [28]).

5.1.3 Modulated Filter Banks and Wavelets

We formulated and developed a complete theory of a special class of filter banks that are easy to design and implement. The M filters in this filter bank are obtained as cosine modulates of a prototype filter. A complete parameterization of such filter banks has been obtained. Wavelets associated with these filter banks have also been characterized. The advantage of these wavelet bases is that the scaling function uniquely determines the wavelets (i.e., there is no need to use a state-space technique to generate wavelets) [14,21,16]

5.1.4 Unitary FIR filter banks with symmetry

In image processing applications, the filters in a filter bank are required to be linear-phase. Moreover, one can impose various symmetry restrictions on the filters (like linear phase). For a number of symmetry classes, a complete parameterization of unitary filter banks and associated wavelet tight frames have been obtained. [20]

5.1.5 Optimal and Robust Multiresolution and Sampling

This research focused on developing the theory and algorithms for obtaining an optimal wavelet multiresolution analysis for the representation of a given signal at a predetermines scale in a variety of error norms [25,34]. Moreover, for classes of signals, the theory and algorithms was extended to permit the designing of a *robust* wavelet multiresolution analysis. All results were derived for the most general case of a M-band multiresolution analysis for arbitrary L^p error norms. An efficient numerical scheme was derived for the design of the optimal wavelet multiresolution analysis when

the least-squared error criterion is used. An important corollary of the analysis is the wavelet sampling theorem, which says that the Nyquist rate samples of a bandlimited signal and the scaling expansion coefficients at a prescribed scale contain the same amount of information (despite the scaling function not being bandlimited). Another corollary is that bandlimited signals are essentially scale limited [34]. Explicit algorithms for the computation of the higher level wavelet coefficients in terms of scaling function coefficients is also obtained [24].

5.1.6 Oversampling Invariance of Wavelet Frames

Given a wavelet frame, if one oversamples by considering not just integer translates, but fractional integer translates, one gets a new set of functions. We have characterized conditions on the oversampling factor that are necessary and sufficient for the new redundant set of functions to form a wavelet frame. Redundancy is desirable since it gives robustness to numerical errors [15].

5.1.7 Completion Problem for Filter Banks

In the design of an M-channel filter bank, sometimes application requirements specify a subset, say L of the M filters. A natural question is what are the conditions on the L filters such that they can be complemented with $M - L$ filters to give rise to a perfect reconstruction M-channel filter bank. Necessary and sufficient conditions for such completions, along with a parameterization of the $M - L$ filters has been obtained for FIR and IIR filter banks

5.1.8 Fundamental tools for Multirate Signal Analysis

Classically multidimensional filter banks have been constructed using a tensor product of one dimensional filter banks. A number of problem arise in the analysis of non-separable multidimensional filter banks, all of which can be traced to the fact that uniform sampling in multiple dimensions is on lattices that are governed by integer matrices. Overcoming this problem, a complete set of tools for the analysis of multidimensional multirate systems has been developed. Using these results, the multidimensional rational sampling rate filter bank problem has been reduced to a multidimensional uniform sampling rate filter bank problem. [19].

5.1.9 State-space approach to wavelets

In the construction of M-band wavelets from filter banks, the scaling function is first constructed, and from it the wavelets. We introduced a novel state-space approach to the construction of

wavelets (from the scaling function). This is the most efficient way to construct wavelets for compactly supported wavelet tight frames [13].

5.1.10 Wavelet-Galerkin Approximation of Differential Operators

Wavelets give a good discrete representation of differential operators (see [1,31,32,23]), and they also give good approximation of "smooth" analog filters. As is shown in the latter two papers in two different manners the degree of approximation is directly related to the length of the scaling vector of the wavelet system.

5.1.11 Wavelet Based Lowpass/Bandpass Interpolation

Orthonormal wavelet bases can be used for efficient lowpass/bandpass interpolation, with the low-pass interpolation exact for polynomials of arbitrary large degree by suitable choice of the wavelet [17]. Moreover, the natural wavelet interpolation at a given level in terms of approximating scaling coefficients by sample values converges in the H^1 norm to a given smooth function which is important for applications to numerical solutions of differential equations (see [40])

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